Math 1-B (Calculus) **Mid-Term Exam-I** **March 28, 2016**

[1]Find **y`** : (4 marks)

$(a)\ y = \cosh x + \sinh \ln x$ $(b)\ y = \sin^{-1} x^2 + (\tanh x)^{-1}$

$(c)\ y = \tanh^{-1} x + 3(\cosh x)^x$ $(d)\ y = t\,\text{sech}\, t\,,\ \ x = 2^t + t^2$

[2]Show that: (2 marks) $\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x}$

[3]Find the integrals: (10 marks)

$(a)\int (3^x + \frac{x}{1+3x^2})\,dx \quad (b)\int (\tanh x + \frac{\cosh \ln x}{x})\,dx$

$(c)\int \frac{x+2}{x^2-4x-5}\,dx \qquad (d)\int \sinh^{-1} x\,dx \qquad (e)\int tan^3 x\,dx$

Answer

$[1](a)\ y = cosh\,x + sinh\,ln\,x \qquad y` = sinh\,x + cosh\,ln\,x\,.\frac{1}{x}$

$(b)\ y = sin^{-1} x^2 + (tanh\,x)^{-1}, \quad y` = \frac{2x}{\sqrt{1-x^4}} - (tanh\,x)^{-2}.\,sech^2 x$

$(c)\ y = \tanh^{-1} x + 3(\cosh x)^x = \tanh^{-1} x + 3e^{x \ln \cosh\ x}$

$$y` = \frac{1}{1-x^2} + 3e^{x \ln \cosh\ x}[1.\ln\cosh x + x\frac{\sinh x}{\cosh x}]$$

$(d)\ y = t\,\text{sech}\,t\,,\ \ x = 2^t + t^2 \qquad y` = \frac{\dot{y}}{\dot{x}} = \frac{1.\,\text{sech}\,t - t\,\text{sech}\,t\,.\tanh t}{2^t \ln 2 + 2t}$

-- ***(4 Marks)***

[2] Show that: $\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x}$

If $y = \tanh^{-1}$ x. Then $\tanh y = \frac{\sinh\ y}{\cosh\ y} = \frac{e^y - e^{-y}}{e^y + e^{-y}} = x$

Then $\frac{e^{2y}-1}{e^{2y}+1} = x$ and $e^{2y} - 1 = x(e^{2y}+1) = x.e^{2y} + x$

Then $e^{2y}(1-x) = 1 + x$. Then $e^{2y} = \frac{1+x}{1-x}$. Then $2y = \ln\frac{1+x}{1-x}$. Then $y = \frac{1}{2}\ln\frac{1+x}{1-x}$.

Then $\tanh^{-1} x = \frac{1}{2}\ \ln\frac{1+x}{1-x}$.

-- ***(2 Marks)***

[3] $(a) \int \left(3^x + \frac{x}{1+3x^2}\right) dx = \frac{3^x}{\ln 3} + \frac{1}{6}\ln(1+3x^2) + c$

$(b) \int (\tanh x + \frac{\cosh \ln x}{x}) \, dx = \ln \cosh x + \sinh \ln x + c$

$(c) \int \frac{x+2}{x^2-4x-5} dx = \int (\frac{A}{x-5} + \frac{B}{x+1}) \, dx = \frac{7}{6}\ln(x-5) - \frac{1}{6}\ln(x+1) + c$

$(d) \int \sinh^{-1} x \, dx = x.\sinh^{-1} x - \int \frac{x}{\sqrt{1+x^2}} dx = x.\sinh^{-1} x - \sqrt{1+x^2} + c$

$(e)\ I_3 = \int tan^3 x \, dx = \frac{1}{2} tan^2 x - I_1 = \frac{1}{2} tan^2 x + \ln \cos x + c$

-- ***(10 Marks)***

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[1]Find **y\` :** (4 marks)

$(a)\ y = \cosh x^2 + \tanh \sin x$ $\qquad (b)\ y = \sinh^{-1} x^3 + (\cosh^{-1} x)^{-1}$

$(c)\ y = \tan^{-1} 2x - 2(\tanh x)^x$ $\qquad (d)\ y = t + \sec t\,,\ x = 3^t . \cosh t$

[2]Show that: (2 marks) $\operatorname{sech}^{-1} x = \ln \frac{1+\sqrt{1-x^2}}{x}$

[3]Find the integrals : (10 Marks)

$(a) \int (e^{3x} + \frac{x}{\sqrt{1+3x^2}})\, dx \qquad (b) \int (\tan x + \frac{\sinh \sqrt{x}}{\sqrt{x}})\, dx$

$(c) \int \frac{x+1}{x^3+x} dx \qquad (d) \int \tan^{-1} x\, dx \qquad (e) \int sin2x.cos^2 3x\, dx$

Answer

$[1](a)\ y = \cosh x^2 + \tanh sin\, x \qquad y` = \sinh x^2 . 2x + sech^2 \sin x . \cos x$

$(b)\ y = \sinh^{-1} x^3 + (\cosh^{-1} x)^{-1} \qquad y` = \frac{3x^2}{\sqrt{1+x^6}} - (\cosh^{-1} x)^{-2} . \frac{1}{\sqrt{x^2-1}}$

$(c)\ y = \tan^{-1} 2x - 2(\tanh x)^x = \tan^{-1} 2x - 2e^{x \ln \tanh\, x}$

$$y` = \frac{2}{1+4x^2} - 2e^{x \ln \tanh\, x}[1. \ln \tanh x + x \frac{sech^2 x}{\tanh x}]$$

$(d)\ y = t + \sec t\,,\ x = 3^t . \cosh t \qquad y` = \frac{\dot{y}}{\dot{x}} = \frac{1 + \sec t . \tan t}{3^t \ln 3 . \cosh t + 3^t . \sinh t}$

-- ***(4 Marks)***

[2]Show that: $\operatorname{sech}^{-1} x = \ln \frac{1+\sqrt{1-x^2}}{x}$

If $y = \operatorname{sech}^{-1} \mathrm{x}$. Then $x = \operatorname{sech} \mathrm{y} = \frac{1}{\cosh\ \mathrm{y}} = \frac{2}{\mathrm{e^y + e^{-y}}}$

Then $x = \frac{2\mathrm{e^y}}{\mathrm{e^{2y}+1}}$ and $x(\mathrm{e^{2y}} + 1) = 2\mathrm{e^y}$.

Then $x\mathrm{e^{2y}} - 2\mathrm{e^y} + \mathrm{x} = 0$. It is second degree equation in $\mathrm{e^y}$.

Then $\mathrm{e^y} = \frac{2 \pm \sqrt{4-4\mathrm{x}^2}}{2\mathrm{x}} = \frac{1 \pm \sqrt{1-\mathrm{x}^2}}{\mathrm{x}}$. Then $\mathrm{y} = \ln \frac{1+\sqrt{1-\mathrm{x}^2}}{\mathrm{x}}$.

-- ***(2 Marks)***

$$[3](a)\int\left(e^{3x}+\frac{x}{\sqrt{1+3x^2}}\right)dx=\frac{e^{3x}}{3}+\frac{1}{6}\frac{\sqrt{1+3x^2}}{1/2}+c$$

$$(b)\int(\tan x+\frac{\sinh\sqrt{x}}{\sqrt{x}})\,dx=-\ln\cos x+2\cosh\sqrt{x}+c$$

$$(c)\int\frac{x+1}{x^3+x}dx=\int(\frac{A}{x}+\frac{Bx+C}{x^2+1})\,dx=\int(\frac{1}{x}+\frac{-x+1}{x^2+1})\,dx=\ln x-\frac{1}{2}\ln(x^2+1)+\tan^{-1}x+c$$

$$(d)\int\tan^{-1}x\,dx=x.\tan^{-1}x-\int\frac{x}{1+x^2}dx=x.\tan^{-1}x-\frac{1}{2}\ln(x^2+1)+c$$

$$(e)\int sin2x.cos^2 3x\,dx=\frac{1}{2}\int sin2x(1+\cos 6x)\,dx=\frac{1}{2}\int(sin2x+\sin 2x\,.\cos 6x)\,dx$$

$$=\frac{1}{2}\int[sin2x+\frac{1}{2}(\sin(-4x)+\sin 8x)]\,dx=\frac{1}{2}\left[\frac{-\cos 2x}{2}+\frac{1}{2}\left(\frac{\cos 4x}{4}-\frac{\cos 8x}{8}\right)\right]+c$$

-- ***(10 Marks)***

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